# PASSPORT FOUR
## ANSWERS

<table>
<thead>
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<th>TOPICS</th>
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</table>
| **1) Reverse Percentages** | Multiplier – 0.86  
Normal Price  
\[
\frac{326.80}{0.86} = \£380
\] | **7) Simultaneous Equations** | b) \[25x - 10y = 180 \]  
6x - 10y = 66  
19x = 114  
x = 6  
5(6) - 2y = 36  
30 - 36 = 2y  
y = -3 |
| **2) Compound interest** | Multiplier – 0.88  
3 Bounces = 2 \times 0.88^3 = 1.36m  
8 Bounces = 2 \times 0.88^8 = 0.72m  
10 Bounces = 2 \times 0.88^{10} = 0.56m | **8) Factorising Quadratics** | a) \((x - 3)(x - 2)\)  
b) \((x + 3)(x - 3)\) |
| **3) Upper and Lower Bounds** | Length 315 \(\leq l < 325\)  
Width 127.5 \(\leq w < 128.5\)  
Perimeter  
\[
315 + 127.5 + 315 + 127.5 = 885m
\] | **9) Changing the Subject** | a) \(bx = p - t\)  
\(x = \frac{p-t}{b}\)  
b) \(\frac{s}{p} = s + q\)  
\(x = p(s + q)\)  
c) \(tx - tm = y + m\)  
\(tx = y + m + tm\)  
\(x = \frac{y + m + tm}{t}\) |
| **4) Error Intervals** | Error interval  
\[8.25 \leq x < 8.35\] | **10) Composite Functions** | 1) \(f g(x) = f(x^2 - 2)\)  
\[
= 4x^2 - 8 + 5 = 4x^2 - 3
\]  
2) \(g f(x) = g(4x + 5)\)  
\[
= 16x^2 + 40x + 25 - 2\]  
\[
= 16x^2 + 40x + 23\] |
| **5) Direct Proportion** | \[y \propto x^2\] so \(y = kx^2\)  
400 = \(k \times 10^2\)  
400 = 100k  
\[
k = \frac{400}{100} = 4
\]  
so \(y = 4x^2\)  
\(y = 4 \times 5^2 = 100\) | **11) Iterative Processes** |  
\[
x_2 = \frac{01^{3} - 3}{4} = \frac{-2}{4} = -\frac{1}{2}
\]  
\[
x_3 = \frac{\left(-\frac{1}{2}\right)^{3} - 3}{4} = -0.78125
\] |
| **6) Calculating with Surds** | 1) \(\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}\)  
\(\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}\)  
\(5\sqrt{3} - 2\sqrt{3} = 3\sqrt{3}\)  
2) \(9 + 3\sqrt{7} - 3\sqrt{7} - \sqrt{49}\)  
\(= 9 - 7 = 2\) | **12) Quadratic Sequences** | 1) \(n^2 + n\)  
2) \(3n^2 - 2n + 4\) |
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| 13) Bearings | Interior Angles $= 180 - 67 = 113^\circ$
Bearing $B$ to $A = 360 - 113 = 247^\circ$
| 19) Frequency Trees | $P(LH) = \frac{3}{8} \times \frac{5}{8} = \frac{15}{64}$
$P(HL) = \frac{5}{8} \times \frac{3}{8} = \frac{15}{64}$
$P(Different) = \frac{15}{64} + \frac{15}{64} = \frac{30}{64}$ |
| 14) Speed, Distance and Time | $6 + 28 = 34$ miles total
10:00am to 10:40am is 40 mins
60mph is 1 mile per min.
28 miles is covered in 28 mins
$40 - 28 = 12$ mins left
$s = \frac{6}{12} = 0.5$ mile per min
30 miles per hour |
| 20) Independent Events | 1) Students $54 + 48 + 16 + 22 = 140$
2) $P(right\,handed\,male) = \frac{48}{140} = \frac{12}{35}$ |
| 15) Volume of Prisms | Break into two rectangles.
CSA= $(2 \times 3) + (4 \times 3) = 18cm^2$
or
CSA=(2 x 6) + (2 x 3) = 18cm^2
Volume = CSA x 4
$= 18 \times 4 = 72cm^3$
| 21) Venn Diagrams | Class 1 scored higher on average
on the test because their median is 80% and Class 2 median is 74%.
Class 2’s results are less consistent because their IQR is (90-70=20) and Class1’s IQR is (85-70=15)
| 16) Arc Length | $\frac{120}{360} \times \pi \times 8 = \frac{8}{3} \pi = 8.38cm$
| 22) Cumulative Frequency |
| 17) Area of a Sector | $\frac{134}{380} \times \pi \times 6^2 = \frac{67}{5} \pi = 42.1cm^2$
| 23) Box Plots |
| 18) Transformations | 1) Translation by $(-3, -6)$
2) Coordinates $(1,4), (-5,4), (-52)\, and\, (-2,1)$ |
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<tr>
<td><strong>Index Notation</strong></td>
<td></td>
<td><strong>Reverse Percentage</strong></td>
<td>$16.12 \div 0.55 = £29.31$</td>
</tr>
<tr>
<td>a) $1$</td>
<td></td>
<td>a) $\sqrt{4 \times 6} = 2\sqrt{6}$</td>
<td></td>
</tr>
<tr>
<td>b) $5 \times 5 \times 5 = 125$</td>
<td></td>
<td>b) $\sqrt{35}$</td>
<td></td>
</tr>
<tr>
<td>c) $\sqrt{64} = 8$</td>
<td></td>
<td>c) $\sqrt{9} + 4\sqrt{3} - 2\sqrt{3} - 6$</td>
<td>$= 3 + 2\sqrt{3} - 6$</td>
</tr>
<tr>
<td>d) $\frac{1}{37} = \frac{1}{2187}$</td>
<td></td>
<td>$= 2\sqrt{3} - 3$</td>
<td></td>
</tr>
<tr>
<td><strong>Percentage Decrease</strong></td>
<td><strong>Percentage Change</strong></td>
<td>$\frac{\text{change}}{\text{original}} \times 100$</td>
<td>$\frac{250,000}{1,500,000} \times 100 = 16.7%$</td>
</tr>
<tr>
<td>$0.96 \times 2500 = £2400$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Recurring Decimals</strong></td>
<td><strong>Surds</strong></td>
<td>a) $\sqrt{4} \times 6 = 2\sqrt{6}$</td>
<td></td>
</tr>
<tr>
<td>a) $\frac{1}{3}$</td>
<td>b) $\sqrt{35}$</td>
<td>b) $\sqrt{9} + 4\sqrt{3} - 2\sqrt{3} - 6$</td>
<td>$= 3 + 2\sqrt{3} - 6$</td>
</tr>
<tr>
<td>b) $\frac{76}{99}$</td>
<td>c) $\sqrt{9} + 4\sqrt{3} - 2\sqrt{3} - 6$</td>
<td>$= 3 + 2\sqrt{3} - 6$</td>
<td>$= 2\sqrt{3} - 3$</td>
</tr>
<tr>
<td>c) $\frac{428}{999}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Fractions and Percentages</strong></td>
<td><strong>Subtracting and Multiplying Mixed Numbers</strong></td>
<td>a) $\frac{19}{5} - \frac{9}{4} = \frac{76}{20} - \frac{45}{20} = \frac{31}{20}$</td>
<td>$= 1 \frac{11}{20}$</td>
</tr>
<tr>
<td>15% in school</td>
<td>$\frac{7}{20} = \frac{35}{100} = 35%$</td>
<td>b) $\frac{13}{3} \times \frac{23}{4} = \frac{299}{12}$</td>
<td>$= 24 \frac{11}{12}$</td>
</tr>
</tbody>
</table>
# Algebra

## Topics

### Midpoint of Coordinates

| X coordinate | \[
\frac{-4 + 10}{2} = \frac{6}{2} = 3
\] |
| Y Coordinate | \[
\frac{6 + (-8)}{2} = \frac{-2}{2} = -1
\] |
| Z Coordinate | \[
\frac{10 + 6}{2} = \frac{-16}{2} = 8
\] |

Midpoint \((3, -1, 8)\)

### Solving Inequalities

\[
4 - 8 \leq 2x < 12 - 8 \\
-4 \leq 2x < 4 \\
-2 \leq x < \frac{4}{2} \\
-2 \leq x < 2
\]

### Quadratic Formula

\[
a = 1, b = -4 \text{ & } c = -8
\]

\[
x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (-8)}}{2 \times 1}
\]

\[
x = \frac{4 \pm \sqrt{16 + 32}}{2}
\]

\[
x = \frac{4 \pm \sqrt{48}}{2}
\]

\[
x = \frac{4 + \sqrt{48}}{2} = 5.46
\]

\[
x = \frac{4 - \sqrt{48}}{2} = -1.46
\]

### Simultaneous Equations

\[
12x + 8y = 52 \\
10x - 8y = 36
\]

\[
22x = 88 \text{ so } x = 4
\]

\[
3(4) + 2y = 13 \\
12 + 2y = 13
\]

\[
2y = 1 \text{ } y = \frac{1}{2}
\]

### Factorise Expressions

1) \(4(x + 5)\)

2) \(3y(y + 4)\)

3) \((x + 7)(x - 3)\)

### Equation of a Line parallel

a) \(y = 2x + C\)

\[\text{E.g.}\]

\[y = 2x \text{ or } y = 2x - 1\]

b) \(y = -\frac{1}{2}x + c\)

\[\text{E.g.}\]

\[y = -\frac{1}{2}x \text{ or } y = -\frac{1}{2}x + 4\]

### Trial and Improvement

<table>
<thead>
<tr>
<th>Value</th>
<th>Output</th>
<th>Big/Small</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>35</td>
<td>small</td>
</tr>
<tr>
<td>6</td>
<td>54</td>
<td>big</td>
</tr>
<tr>
<td>5.5</td>
<td>44</td>
<td>small</td>
</tr>
<tr>
<td>5.6</td>
<td>45.92</td>
<td>small</td>
</tr>
<tr>
<td>5.7</td>
<td>47.88</td>
<td>big</td>
</tr>
<tr>
<td>5.65</td>
<td>46.895</td>
<td>small</td>
</tr>
</tbody>
</table>

Answer \(x = 5.7\)

### Change the Subject

1) \[3x = y - t\]

\[x = \frac{y - t}{3}\]

2) \[\frac{x}{p} = z + pr\]

\[x = p(z + pr)\]

3) \[tx + tr = p\]

\[tx = p - tr\]

\[x = \frac{p - tr}{t}\]
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#### Shapes and Measures

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<tbody>
<tr>
<td><strong>Area of a Triangle</strong></td>
<td>Sohcahtoa</td>
<td><strong>Bearings</strong></td>
<td>180 − 75 = 105°</td>
</tr>
<tr>
<td></td>
<td><em>opp = 11 × sin(67)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 10.1cm <em>(height of triangle)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>adj = 11 × cos(67)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 4.298cm <em>(half base)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Full base = 8.596cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Area 10.1 × 4.298 = 43.41cm²</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Pythagoras’ Theorem</strong></td>
<td><em>x = √13² − 9²</em></td>
<td><strong>Angles in Polygons</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>x = 9.38cm</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Volume of a Cylinder</strong></td>
<td><em>V = π × 2.5² × 9</em></td>
<td><strong>Perimeter of a Sector</strong></td>
<td>360 − 72 = 288</td>
</tr>
<tr>
<td></td>
<td>= 176.7cm³</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Transformation – Enlargement from a point</strong></td>
<td><em>Enlargement Scale Factor -1</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Centre (0 , 0)</td>
<td></td>
<td></td>
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Frequency Polygon

Statistics

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<tr>
<td><strong>Frequency Polygon</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td><strong>Mean from a table</strong></td>
<td></td>
</tr>
<tr>
<td>Height (cm)</td>
<td>Frequency</td>
</tr>
<tr>
<td>0 &lt; h ≤ 10</td>
<td>9</td>
</tr>
<tr>
<td>10 &lt; h ≤ 20</td>
<td>7</td>
</tr>
<tr>
<td>20 &lt; h ≤ 40</td>
<td>8</td>
</tr>
<tr>
<td>40 &lt; h ≤ 50</td>
<td>6</td>
</tr>
<tr>
<td><strong>Box Plots</strong></td>
<td></td>
</tr>
<tr>
<td>Golf club A members are older because their median age is 52 and golf club B’s median age is 49. Golf club A’s age range is larger and less consistent because their IQR is (59-29=30) and gold club B’s IQR is (59-49=10)</td>
<td></td>
</tr>
<tr>
<td><strong>Cumulative Frequency</strong></td>
<td></td>
</tr>
<tr>
<td>LQ is 11 mins (read at cf 25)</td>
<td></td>
</tr>
<tr>
<td>Median is 14 mins (read at cf 50)</td>
<td></td>
</tr>
<tr>
<td>UQ is 18 mins (read at cf 75)</td>
<td></td>
</tr>
<tr>
<td>IQR = 18 − 11 = 7 mins</td>
<td></td>
</tr>
<tr>
<td><strong>Expected Probability</strong></td>
<td></td>
</tr>
<tr>
<td>$\frac{2}{5}$ of $150 = \frac{150}{5} \times 2 = 30 \times 2 = 60$ games</td>
<td></td>
</tr>
<tr>
<td><strong>Sample Space</strong></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Head</td>
<td>1H</td>
</tr>
<tr>
<td>Tail</td>
<td>1T</td>
</tr>
</tbody>
</table>

$P(H, \text{Even}) = \frac{3}{12} = \frac{1}{4}$