

# PASSPORT FIVE

## ANSWERS

TOPICS	ANSWERS	TOPICS	ANSWERS
<b>1) Area Underneath a Curve</b>	$\text{Area A} = 10 \times 20 \div 2 = 100m$ $\text{Area B} = 20 \times 20 = 400m$ $\text{Area C} = 40 \times 20 \div 2 = 400m$ $\text{Total Distance} = 100 + 400 + 400 = 900m$	<b>5) Change the Subject</b>	$a) \begin{aligned} ax + ap &= bx + t \\ ax - bx &= t - ap \\ x(a - b) &= t - ap \\ x &= \frac{t - ap}{a - b} \end{aligned}$ $b) \begin{aligned} p(x + t) &= y(x + t) \\ px + pt &= yx + yt \\ px - yx &= yt - pt \\ x(p - y) &= yt - pt \\ x &= \frac{yt - pt}{p - y} \end{aligned}$
<b>2) Inverse Proportion</b>	$y \propto \frac{1}{x^2} \text{ so } y = \frac{k}{x^2}$ $18 = \frac{k}{3^2}$ $18 = \frac{k}{9}$ $k = 18 \times 9 = 162$ $y = \frac{162}{x^2}$	<b>6) Simultaneous Equations by substitution</b>	$a) \begin{aligned} y &= 3 - x \\ \text{so } x^2 + (3 - x)^2 &= 5 \\ 2x^2 - 6x + 4 &= 0 \\ x^2 - 3x + 2 &= 0 \\ (x - 2)(x - 1) &= 0 \\ x_1 &= 1 \text{ and } x_2 = 2 \\ y_1 &= 3 - 1 = 2 \text{ and } y_2 = 3 - 2 = 1 \end{aligned}$ $b) \begin{aligned} 4x^2 + 4 &= 2x + 4 \\ x_1 &= \frac{1}{2} \text{ and } x_2 = 0 \\ y_1 &= 5 \text{ and } y_2 = 4 \end{aligned}$
<b>3) Rationalising Surds</b>	$1) \frac{5}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{5\sqrt{7}}{7}$ $2) \frac{2\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{15}}{5}$ $3) \frac{\sqrt{6}}{4+3\sqrt{2}} \times \frac{4-3\sqrt{2}}{4-3\sqrt{2}} = \frac{4\sqrt{6}-3\sqrt{12}}{16-9\sqrt{4}}$ $= \frac{4\sqrt{6}-3 \times 2\sqrt{3}}{16-18}$ $= \frac{4\sqrt{6}-6\sqrt{3}}{-2}$	<b>7) Algebraic Proof</b>	<p>Consecutive even numbers <math>2n</math> and <math>2n + 2</math></p> $\begin{aligned} (2n)^2 + (2n + 2)^2 &= 4n^2 + 4n^2 + 8n + 4 \\ &= 8n^2 + 8n + 4 \\ &= 4(2n^2 + 2n + 1) \end{aligned}$ <p>A factor of 4 can be taken out front. Anything multiplied by 4 must be in the 4 times table and therefore a multiple of 4.</p>
<b>4) Solving Quadratics</b>	$1) (x + 4)(x + 3) = 0$ $x = -4 \text{ and } x = -3$ $2) (2x + 3)(x - 5) = 0$ $x = -\frac{3}{2} \text{ and } x = 5$	<b>8) Identities</b>	$3ax + 7 + 5x - b \equiv 2 - x$ $3ax + 5x = -x$ <p>so <math>a = -2</math> because <math>(-6x + 5x = -x)</math></p> $7 - b = 2$ $7 - 5 = 2$ <p>So <math>b = 5</math></p>

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9) 3D Trigonometry	<p>Base Hypotenuse</p> $h = \sqrt{20^2 + 8^2} = 4\sqrt{29}$ $= 21.5407m$ <p>Height of may pole OP</p> $\text{Tan}(50) = \frac{\text{Opp}}{4\sqrt{29} \div 2}$ $\text{Opp} = 2\sqrt{29} \times \text{Tan}(50)$ $\text{Opp} = 12.84m$	13) Probability of Independent Events	$P(YY) = \frac{10}{24} \times \frac{9}{23} = \frac{90}{552} = \frac{15}{92}$
10) Enlargement by a negative scale factor	<p>Coordinates (4,2), (1,2) and (1,5)</p>	14) Interpreting Histograms	<p>a) <math>30 + (45 + 6) + (15 + 9) = 105</math> people</p> <p>b) <math>15 + (45 + 6) + (15 + 9) = 90</math> people</p> <p>c) 55 to 65 miles  <math>(15 + 9) + (45 + 12) = 81</math>              Didn't travel 55 to 65 miles              30 to 55 miles is <math>105 + 24 = 129</math> people              65 to 70 miles is 39 people              Altogether  <math>39 + 129 = 168</math> people</p>
11) Vectors	$\overrightarrow{AB} = -3a + 2b - a$ $= 2b - 4a$ $\overrightarrow{AM} = \frac{1}{2}(2b - 4a)$ $= b - 2a$	15) Reverse Averages	<p>a) __, __, 7, __, __ median 7</p> <p>b) 5, 5, 7, __, __ mode 5              Range of 4              5,5,7,8,9</p>
12) Circle Theorems	<p>Cyclic Quadrilateral.              Opposite angles add to make 180 degrees.</p> $x = 180 - 75 = 105^\circ$		

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## ANSWERS NUMBER

TOPICS	ANSWERS	TOPICS	ANSWERS												
Index Notation	$a) 1$ $b) (\sqrt[3]{125})^2 = 5^2 = 25$ $c) \sqrt{64} = 8$ $d) \frac{1}{3^2} = \frac{1}{9}$	Reverse Percentage	$16.12 \div 0.55 = \text{£}29.31$												
Successive Percentage Change and Compound Growth and Decay.	$2500 \times 0.9 = \text{£}2250$ $2250 \times 0.95^3 = \text{£}1929.09$	Bounds	<table border="1"> <thead> <tr> <th></th> <th>Min</th> <th>Max</th> </tr> </thead> <tbody> <tr> <td>L</td> <td>7.45cm</td> <td>7.55cm</td> </tr> <tr> <td>W</td> <td>11.5cm</td> <td>12.5cm</td> </tr> <tr> <td>Area</td> <td><math>85.8\text{cm}^2</math></td> <td><math>94.4\text{m}^2</math></td> </tr> </tbody> </table>		Min	Max	L	7.45cm	7.55cm	W	11.5cm	12.5cm	Area	$85.8\text{cm}^2$	$94.4\text{m}^2$
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Recurring Decimals	$a) \frac{7}{9}$ $b) \frac{758}{999}$ $c) \frac{542}{990}$	Surds	$a) \sqrt{4 \times 6} = 2\sqrt{6}$ $b) \sqrt{35}$ $c) \sqrt{9} + 4\sqrt{3} - 2\sqrt{3} - 6 = 3 + 2\sqrt{3} - 6 = 2\sqrt{3} - 3$												
Fractions and Percentages	$T \propto m$ so $T = km$ $600 = 250k$ $k = \frac{600}{250} = 2.4$ $T = 2.4m$ $T = 2.4 \times 400 = 960$ seconds	Surds – Rationalise the denominator	$a) \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$ $b) \frac{2}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{6+2\sqrt{5}}{9-5} = \frac{6+2\sqrt{5}}{4} = \frac{3+\sqrt{5}}{2}$												

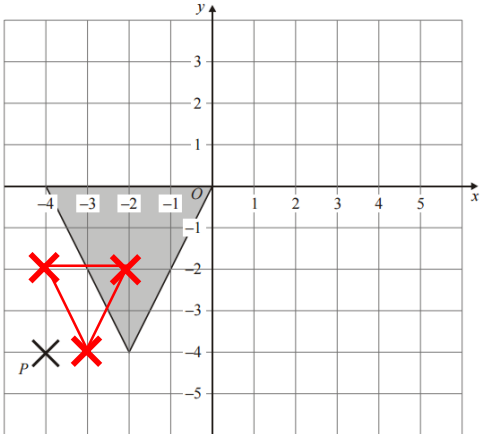
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### Algebra

TOPICS	ANSWERS	TOPICS	ANSWERS
Midpoint of Coordinates	X coordinate $\frac{-4 + 10}{2} = \frac{6}{2} = 3$ Y Coordinate $\frac{6 + (-8)}{2} = -\frac{2}{2} = -1$ Z Coordinate $\frac{10 + 6}{2} = \frac{16}{2} = 8$ Midpoint (3, -1, 8)	Completing the Square	$(x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$ $p = 3 \text{ and } q = -11$
Quadratic Formula	$a = 1, b = -4 \text{ \& } c = -8$ $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (-8)}}{2 \times 1}$ $= \frac{4 \pm \sqrt{16 + 32}}{2}$ $x = \frac{4 + \sqrt{48}}{2} = 5.46$ $x = \frac{4 - \sqrt{48}}{2} = -1.46$	Simultaneous Equations	$x^2 - 1 = 5x - 1$ $x^2 - 5x = 0$ $x(x - 5) = 0$ $x_1 = 0 \text{ and } x_2 = 5$ $y_1 = -1$ $y_2 = 25 - 1 = 24$
Factorise Expressions	1) $4(x + 5)$ 2) $3y(y + 4)$ 3) $(x + 7)(x - 3)$	Equation of parallel and perpendicular lines	a) $y = 2x + c$ $4 = 2(3) + c$ $4 = 6 + c \text{ so } c = -2$ $y = 2x - 2$ b) $y = -\frac{1}{2}x + c$ $5 = -\frac{1}{2}(-2) + c$ $5 = 1 + c \text{ so } c = 4$ $y = -\frac{1}{2}x + 4$
Algebraic Manipulation	$LHS = \frac{4(a^3 - a)}{(a + a^2)(ab)}$ $= \frac{4a^3 - 4a}{a^2b + a^3b}$ $= \frac{4a(a^2 - 1)}{a^2b(1 + a)} = \frac{4(a^2 - 1)}{ab(1 + a)}$ $= \frac{4(a + 1)(a - 1)}{ab(a + 1)}$ $= \frac{4(a - 1)}{ab} = RHS$	Change the Subject	$y(x - a) = x + 2a$ $yx - ya = x + 2a$ $yx - x = 2a + ya$ $x(y - 1) = 2a + ya$ $x = \frac{2a + ay}{y - 1}$

**PASSPORT FIVE**  
**ANSWERS**  
**Shapes and Measures**

TOPICS	ANSWERS	TOPICS	ANSWERS
<b>Volume of Frustum</b>	<p>Vol big cone</p> $\frac{1}{3}(\pi \times 15^2 \times 40) = 3000\pi$ <p>Vol small cone Radius = 7.5cm</p> $\frac{1}{3}(\pi \times 7.5^2 \times 20) = 375\pi$ <p>Vol Frustum</p> $3000\pi - 375\pi = 2625\pi$ $= 8246.68\text{cm}^3$	<b>Trigonometry</b> <b>Area of a Triangle</b>	$\frac{1}{2}(12 \times 14 \times \sin(50))$ $= 64.3\text{cm}^2$
<b>3D Pythagoras' Theorem</b>	$BD = \sqrt{8^2 + 4^2} = 4\sqrt{5}$ $= 8.944\text{cm}$ $DF = \sqrt{(4\sqrt{5})^2 + 8^2}$ $= 12\text{cm}$	<b>Similar Shapes</b>	<p>Length SF = <math>\frac{40}{50} = \frac{4}{5}</math></p> <p>Area SF = <math>\left(\frac{4}{5}\right)^2 = \frac{16}{25}</math></p> $3.27 \times \frac{16}{25}$ $= 2.0928\text{m}^2$
<b>Circle Theorem</b>	$AOB = 180 - (90 + 36)$ $= 54^\circ$ <p>Tangent meets at right angle to centre.</p> $ABC = 27^\circ$ <p>The angle at the centre is twice the angle at the circumference (from the same chord)</p>	<b>Perimeter of a Sector</b>	$360 - 72 = 288$ $\left(\frac{288}{360} \times \pi \times 18\right) + 9$ $+ 9$ $= 63.24\text{ cm}$
<b>Transformation – Enlargement from a point</b>	<p><b>Coordinates</b></p> <p><math>(-4, -2)</math>, <math>(-2, -2)</math> and <math>(-3, -4)</math></p>		

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### Statistics

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Algebra Probability	$20x = 1$ so $x = \frac{1}{20}$ $Chocolate = \frac{3}{20}$ $Bubble Bath = \frac{8}{20} = \frac{2}{5}$ $Toys = \frac{4}{20} = \frac{1}{5}$ $Other = \frac{5}{20} = \frac{1}{4}$	Independent Probability	$P(BB) = \frac{5}{7} \times \frac{4}{6} = \frac{20}{42}$ $P(GG) = \frac{2}{7} \times \frac{1}{6} = \frac{2}{42}$ $P(Same) = \frac{20}{42} + \frac{2}{42} = \frac{22}{42}$ $= \frac{11}{21}$																					
Mean from a table	<table border="1"> <thead> <tr> <th>Height (cm)</th> <th>Frequency</th> <th>MP</th> <th>Fx</th> </tr> </thead> <tbody> <tr> <td><math>0 &lt; h \leq 10</math></td> <td>9</td> <td>5</td> <td>45</td> </tr> <tr> <td><math>10 &lt; h \leq 20</math></td> <td>7</td> <td>15</td> <td>105</td> </tr> <tr> <td><math>20 &lt; h \leq 40</math></td> <td>8</td> <td>30</td> <td>240</td> </tr> <tr> <td><math>40 &lt; h \leq 50</math></td> <td>6</td> <td>45</td> <td>270</td> </tr> </tbody> </table>	Height (cm)	Frequency	MP	Fx	$0 < h \leq 10$	9	5	45	$10 < h \leq 20$	7	15	105	$20 < h \leq 40$	8	30	240	$40 < h \leq 50$	6	45	270		$Mean = \frac{660}{30}$ $= 22cm$	
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Box Plots	<p>Golf club A members are older because their median age is 52 and golf club B's median age is 49. Golf club A's age range is larger and less consistent because their IQR is <math>(59-29=30)</math> and golf club B's IQR is <math>(59-49=10)</math></p>	Cumulative Frequency	<p>LQ is 11 mins (read at cf 25)                      Median is 14 mins (read at cf 50)                      UQ is 18 mins (read at cf 75)</p> <p><math>IQR = 18 - 11 = 7</math> mins</p>																					
Histogram	$12 \div 10 = 1.2$ $14 \div 20 = 0.7$ $8 \div 20 = 0.4$ $6 \div 10 = 0.6$		<p>40 to 50 cm  <math>10 \times 0.4 = 4</math> objects</p> <p>50 to 60 cm  <math>10 \times 0.6 = 6</math> objects</p> <p>12 objects taller than 40cm.</p>																					
Sample Space	<table border="1"> <tbody> <tr> <td></td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>Head</td> <td>1H</td> <td>2H</td> <td>3H</td> <td>4H</td> <td>5H</td> <td>6H</td> </tr> <tr> <td>Tail</td> <td>1T</td> <td>2T</td> <td>3T</td> <td>4T</td> <td>5T</td> <td>6T</td> </tr> </tbody> </table>		1	2	3	4	5	6	Head	1H	2H	3H	4H	5H	6H	Tail	1T	2T	3T	4T	5T	6T		$P(H, Even) = \frac{3}{12} = \frac{1}{4}$
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